Unit 5 Test Study Guide
(Polynomial Functions)

Name: _____

Date: _____ Per: _____

Topic 1: Classifying Polynomials & Polynomial Operations

Simplify each expression. Final answers should be written in standard form.

1. $(-4m^2n)^4 \cdot \frac{1}{6}m^{-10}n^{-4}$

$256m^8n^4 \cdot \frac{1}{6}m^{-10}n^4$

$= \boxed{\frac{128}{3m^2}}$

2. $(8a^2-6-8a)+(1-6a-7a^2)$

$\boxed{a^2-14a-5}$

3. $(6x-7x^2+7)-(5x^2+2x-2x^3-1)$

$\boxed{2x^3-12x^2+4x+8}$

4. $(y+4)^3-2y(y-1)$

$= (y+4)(y+4)(y+4)-2y^2+2y$

$= (y+4)(y^2+8y+16)-2y^2+2y$

$= y^3+8y^2+16y+4y^2+32y$

$+64-2y^2+2y$

$= \boxed{y^3+10y^2+50y+64}$

5. $(3k-6)(k^2-k+7)$

$= 3k^3-3k^2+21k-6k^2$

$+6k-42$

$= \boxed{3k^3-9k^2+27k-42}$

6. $\frac{-8c^6d^4+56c^4d^2-24c^2d}{8c^2d}$

$= \boxed{-c^4d^3+7c^2d-3}$

Topic 2: Factoring Polynomials

Differences of Squares	Sum of Cubes	Differences of Cubes
$a^2-b^2=$	$a^3+b^3=$	$a^3-b^3=$
$(a+b)(a-b)$	$(a+3)(a^2-3a+9)$	$(a-3)(a^2+3a+9)$

Factor each polynomial below completely.

7. $9x^3+21x^2$

$\boxed{3x^2(3x+7)}$

8. $3n^4-147$

$3(n^4-49)$

$\boxed{3(n^2+7)(n^2-7)}$

9. $64a^3-343b^3$

$\boxed{(4a-7b)(16a^2+28ab+49b^2)}$

10. $648w+1029w^4$

$3w(216+343w^3)$

$\boxed{3w(6+7w)(36-42w+49w^2)}$

11. $32c^5d-162cd^3$

$2cd(16c^4-81d^2)$

$\boxed{2cd(4c^2+9d)(4c^2-9d)}$

12. $216pq-p^7q$

$pq(216-p^6)$

$\boxed{pq(6-p^2)(36+6p^2+p^4)}$

13. $2c^5 - 2c^3 - 60c$

$2c(c^4 - c^2 - 30)$

$\boxed{2c(c^2-6)(c^2+5)}$

14. $9y^4 - 7y^2 - 16$

$y^4 - 7y^2 - 144$

$(y^2 - \frac{16}{9})(y^2 + \frac{9}{9})$

$(9y^2-16)(y^2+1)$

$\boxed{(3y+4)(3y-4)(y^2+1)}$

15. $n^3 + 2n^2 - 36n - 72$

$n^2(n+2) - 36(n+2)$

$(n^2-36)(n+2)$

$\boxed{(n+6)(n-6)(n+2)}$

16. $8x^3 - 10x^2 + 28x - 35$

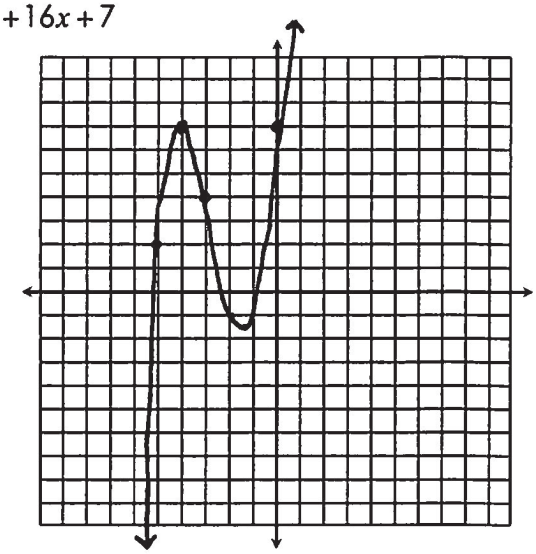
$2x^2(4x-5) + 7(4x-5)$

$\boxed{(2x^2+7)(4x-5)}$

Topic 3: Graphing Polynomial Functions

Graph each function and identify its key characteristics.

17. $f(x) = x^3 + 8x^2 + 16x + 7$



Domain: $\{\mathbb{R}\}$

Range: $\{\mathbb{R}\}$

Rel. Maximum(s): $(-4, 7)$

Rel. Minimum(s): $(-1.34, -2.48)$

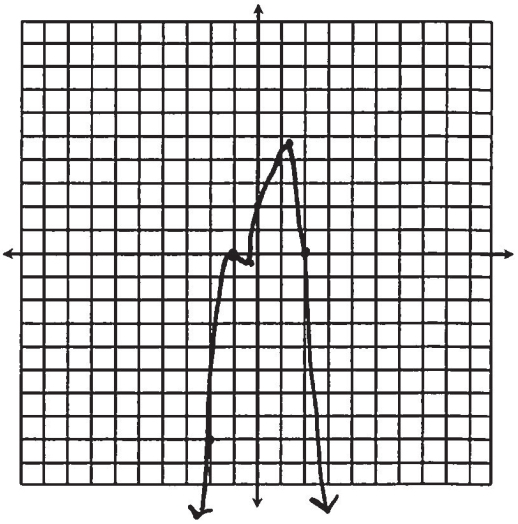
End Behavior: As $x \to -\infty$, $f(x) \to$ $-\infty$

As $x \to \infty$, $f(x) \to$ ∞

Inc. Intervals: $(-\infty, -4)\ (-1.34, \infty)$

Dec. Intervals: $(-4, -1.34)$

18. $f(x) = -x^4 + 3x^2 + 2x$



Domain: $\{\mathbb{R}\}$

Range: $\{y \mid y \leq 4.85\}$

Rel. Maximum(s): $(-1, 0)$, $(1.37, 4.85)$

Rel. Minimum(s): $(-.36, -.35)$

End Behavior: As $x \to -\infty$, $f(x) \to$ $-\infty$

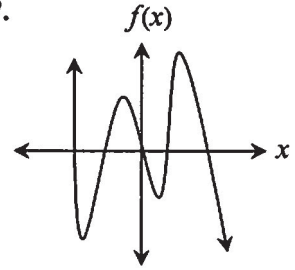
As $x \to \infty$, $f(x) \to$ $-\infty$

Inc. Intervals: $(-\infty, -1)$, $(-.36, 1.37)$

Dec. Intervals: $(-1, -.36)$, $(1.37, \infty)$

Determine the end behavior, whether the function is an even or odd degree, and the sign of the leading coefficient given each graph below.

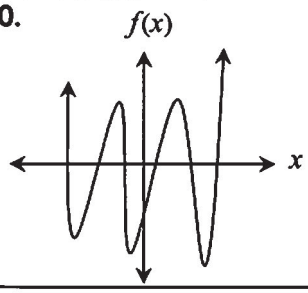
19.



Odd degree,

Negative

20.



Even degree,

Positive

Identify the zeros (and their multiplicity) of each function below and the effect on the graph.

21. $f(x) = 3x^3(x-1)^2(x-8)$

Zero	Multiplicity	Effect
0	3	Crosses
1	2	bounces
8	1	crosses

22.. $f(x) = -x(5x+3)^3(x-2)^8$

Zero	Multiplicity	Effect
0	1	Crosses
$-3/5$	3	crosses
2	8	bounces

23. The graph of a polynomial function has zeros of 0 (multiplicity 2), 1 (multiplicity 2), and 5/2 (multiplicity 2). Write a function in standard form that could represent this function.

$f(x) = x^2(x-1)^2(2x-5)^2$

$f(x) = x^2(x^2-2x+1)(4x^2-20x+25)$

$f(x) = x^2(4x^4-20x^3+25x^2-8x^3+40x^2-50x+4x^2-20x+25)$

$f(x) = x^2(4x^4-28x^3+69x^2-70x+25)$

$f(x) = \boxed{4x^6-28x^5+69x^4-70x^3+25x^2}$

Topic 4: Solving Polynomial Equations

Solve each equation. Simplify all irrational and complex solutions.

24. $2x^4 - 48x^2 = 0$

$2x^2(x^2-24)=0$

$2x^2=0$	$x^2=24$
$x^2=0$	$x=\pm\sqrt{24}$
$x=0$	$x=\pm 2\sqrt{6}$

$\boxed{x=\{0,\pm 2\sqrt{6}\}}$

25. $25x^3 = 64x$

$25x^3-64x=0$

$x(25x^2-64)=0$

$x(5x+8)(5x-8)=0$

$x=0$	$5x+8=0$	$5x-8=0$
	$x=\frac{-8}{5}$	$x=\frac{8}{5}$

$\boxed{x=\{0,\pm\frac{8}{5}\}}$

Topic 5: Dividing Polynomials & The Remainder Theorem

Find each quotient.

32. $(x^3+x^2-71x+9)\div(x+9)$

$$\begin{array}{r} x^2-8x+1 \\ x+9 \overline{)\,x^3+x^2-71x+9} \\ -(x^3+9x^2) \\ \hline -8x^2-71x \\ -(-8x^2-72x) \\ \hline x+9 \\ -(x+9) \\ \hline 0 \end{array}$$

x^2-8x+1

33. $(8n^3-36n^2-15n-16)\div(n-5)$

$$\begin{array}{r|cccc} 5 & 8 & -36 & -15 & -16 \\ & \downarrow & 40 & 20 & 25 \\ \hline & 8 & 4 & 5 & (9) \end{array}$$

$8n^2+4n+5+\frac{9}{n-5}$

34. $(12a^3+2a^2-6a-30)\div(3a-4)$

$$\begin{array}{r} 4a^2+6a+6 \\ 3a-4 \overline{)\,12a^3+2a^2-6a-30} \\ -(12a^3-16a^2) \\ \hline 18a^2-6a \\ -(18a^2-24a) \\ \hline 18a-30 \\ -(18a-24) \\ \hline -6 \end{array}$$

$4a^2+6a+6-\frac{6}{3a-4}$

35. $(y^4+6y^3-4y-31)\div(y+6)$

$$\begin{array}{r|ccccc} -6 & 1 & 6 & 0 & -4 & -31 \\ & \downarrow & -6 & 0 & 0 & 24 \\ \hline & 1 & 0 & 0 & -4 & (-7) \end{array}$$

$y^3-4-\frac{7}{y+6}$

36. Using the Remainder Theorem, find $f(-2)$ when $f(x)=3x^3-28x^2+70$.

$$\begin{array}{r|cccc} -2 & 3 & -28 & 0 & 70 \\ & \downarrow & -6 & 68 & -136 \\ \hline & 3 & -34 & 68 & (-66) \end{array}$$

$f(-2)=-66$

37. The profit P of a small business (in thousands of dollars) since it was founded can be modeled by the function below, where t is the years since 1990. Use the Remainder Theorem to find the company's profit in 2017.

$$P(t)=0.5t^4-3t^3+t^2+25$$

$$\begin{array}{r|ccccc} 27 & .5 & -3 & 1 & 0 & 25 \\ & \downarrow & 13.5 & 283.5 & 7681.5 & 207400.5 \\ \hline & .5 & 10.5 & 284.5 & 7681.5 & 207425.5 \end{array}$$

$\$207,425,500$

26. $108x^3 + 37 = 5$

$108x^3 + 32 = 0$

$4(27x^3 + 8) = 0$

$4 \mid (3x+2)(9x^2 - 6x + 4) = 0$

$4 \neq 0$ | $3x+2=0$, $x = -\frac{2}{3}$ | $x = \frac{6 \pm \sqrt{(-6)^2 - 4(9)(4)}}{2(9)}$

$x = \frac{6 \pm \sqrt{-108}}{18}$

$x = \frac{6 \pm 6i\sqrt{3}}{18}$

$x = \frac{1 \pm i\sqrt{3}}{6}$

$x = \left\{-\frac{2}{3}, \frac{1 \pm i\sqrt{3}}{3}\right\}$

27. $9x^5 - 72x^2 = 0$

$9x^2(x^3 - 8) = 0$

$9x^2 \mid (x-2)(x^2 + 2x + 4) = 0$

$9x^2 = 0$, $x^2 = 0$, $x = 0$ | $x - 2 = 0$, $x = 2$ | $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{-12}}{2}$

$x = \frac{-2 \pm 2i\sqrt{3}}{2}$

$x = -1 \pm i\sqrt{3}$

$x = \{0, 2, -1 \pm i\sqrt{3}\}$

28. $x^4 + 19x^2 - 20 = 0$

$(x^2 + 20)(x^2 - 1) = 0$

$x^2 + 20 = 0$, $x^2 = -20$, $x = \pm i\sqrt{20}$, $x = \pm 2i\sqrt{5}$ | $x^2 - 1 = 0$, $x^2 = 1$, $x = \pm 1$

$x = \{\pm 1, \pm 2i\sqrt{5}\}$

29. $x^5 = 18x^3 - 81x$

$x^5 - 18x^3 - 81x = 0$

$x(x^4 - 18x^2 - 81) = 0$

$x \mid (x^2 - 9)(x^2 - 9) = 0$

$x = 0$ | $x^2 - 9 = 0$, $x^2 = 9$, $x = \pm 3$ | $x^2 - 9 = 0$, $x^2 = 9$, $x = \pm 3$

$x = \{0, \pm 3\}$

30. $3x^4 - 14x^2 = 5$

$3x^4 - 14x^2 - 5 = 0$

$x^4 - 14x^2 - 15 = 0$

$(x^2 - \frac{15}{3})(x^2 + \frac{1}{3}) = 0$

$(x^2 - 5)(3x^2 + 1) = 0$

$x^2 - 5 = 0$, $x^2 = 5$, $x = \pm\sqrt{5}$ | $3x^2 + 1 = 0$, $x^2 = -\frac{1}{3}$, $x = \pm\frac{i}{\sqrt{3}}$

$x = \pm\frac{i\sqrt{3}}{3}$

$x = \left\{\pm\sqrt{5}, \pm\frac{i\sqrt{3}}{3}\right\}$

31. $2x^3 + 7x^2 - 16x - 56 = 0$

$x^2(2x + 7) - 8(2x + 7) = 0$

$(x^2 - 8)(2x + 7) = 0$

$x^2 - 8 = 0$, $x^2 = 8$, $x = \pm 2\sqrt{2}$ | $2x + 7 = 0$, $x = -\frac{7}{2}$

$x = \left\{-\frac{7}{2}, \pm 2\sqrt{2}\right\}$

Topic 6: Operations & Compositions of Functions

Given $f(x) = x^2 + 4x - 12$, $g(x) = 5x^2 - 2$, and $h(x) = x + 7$, find each function. Indiciate any restrictions in the domain.

38. $(f - g)(x)$

$(x^2 + 4x - 12) - (5x^2 - 2)$

$= \boxed{-4x^2 + 4x - 10}$

39. $(h \cdot g)(x)$

$(x+7)(5x^2 - 2)$

$= \boxed{5x^3 + 35x^2 - 2x - 14}$

40. $\left(\frac{h}{f}\right)(x)$

$$\frac{x+7}{x^2+4x-12}$$

$$= \boxed{\frac{x+7}{(x+6)(x-2)} \; ; \{x \mid x \neq -6, 2\}}$$

41. $(g \circ h)(x)$

$5(x+7)^2 - 2$

$= 5(x^2 + 14x + 49) - 2$

$= \boxed{5x^2 + 70x + 243}$

Use the same functions above, evaluate each function.

42. $(g + h)(-4)$

$g(-4) = 5(-4)^2 - 2$

$= 5(16) - 2$

$= 78$

$h(-4) = -4 + 7$

$= 3$

$78 + 3 = \boxed{81}$

43. $(h \circ f)(2)$

$f(2) = (2)^2 + 4(2) - 12$

$= 4 + 8 - 12 = 0$

$h(0) = 0 + 7$

$= \boxed{7}$

Topic 7: Regression

44. The population present in a bacteria culture over 5 days is given in the table below. Write a **cubic function** to represent the data.

Time (days)	0	1	2	3	4	5
Population	28	135	219	332	520	834

$f(x) = 8.18x^3 - 35.41x^2 + 133.83x + 28.08$

45. Use a **cubic function** to estimate the value of y when x is -8. How does the estimate change when a quartic function is used instead?

x	-4	0	4	8	12
y	975	128	-9	-160	-893

Cubic:

$f(x) = -1.68x^3 + 21.14x^2 - 97.48x + 136.91$

$f(-8) = 3129.87$

Quartic:

$f(x) = 0.03x^4 - 2.09x^3 + 21.78x^2 - 89.58x + 128$

$f(-8) = 3431.52$

They differ by 301.65.